SPECTRAL APPROACH TO CALCULATE SPECULAR REFLECTION OF LIGHT FROM WAVY WATER SURFACE

Robert Arnone
University of Southern Mississippi

SPECTRAL APPROACH TO CALCULATE SPECULAR REFLECTION OF LIGHT FROM WAVY WATER SURFACE

V. I. Haltrin,† W. E. McBride III,° and R. A. Arnone †

† Naval Research Laboratory, Ocean Optics Section, Code 7333
Stennis Space Center, MS 39529-5004, USA. e-mail: <haltrin@nrlssc.navy.mil>

° Planning Systems Inc., 115 Christian Lane, Slidell, LA 70458, USA

Abstract

In studying light and image transfer in sea water the influence of Fresnel surface reflection is as significant as scattering and absorption phenomena. In these cases a knowledge of the reflective properties of sea surface at different wind speeds is very important. At present, little is published about these properties. We present here results of numerical modeling of Fresnel light reflection coefficient of sea water as a function of solar zenith angle and wind speed. The ray-tracing computer model was developed to generate wave slopes and elevations. In order to generate a realistic sea surface the model used Pierson-Moskowitz and Paul Hwang wave height spectrums. The final result of this paper is a simple equation and very fast FORTRAN code to calculate Fresnel reflection coefficient of wavy water surface.

Introduction

The idea of the approach used in this paper is to reproduce a realistic wavy sea surface using wave energy spectrums published in literature. To reproduce rough wave surface we computed elevations in a sample square area of sea surface by using the following method: all significant range of wave numbers k of the wave energy spectrum was divided by N intervals. For each interval we computed an amplitude A_k of the wave using energy spectrum $\chi(k)$ taken from the literature. The sample sea surface area was represented as a $M \times M$ grid. The weight of the amplitude in each grid point was computing using Monte Carlo method. In each grid point we averaged amplitudes of randomly generated K flat sine waves. The direction and phase of each flat sine wave was chosen randomly. This procedure gives weights w_{ij} $(0 \le i \le M, 0 \le j \le M)$ for each pixel of sample area in the range of $0 \le w_{ij} \le 1$. The averaging of $w_{ij} A_k$ over energy spectrum $\chi(k)$ gives us an average elevation $\overline{A}_{ij}$ of sea surface in a grid point (i, j). Having elevations $\overline{A}_{ij}$ we can compute elevations and orientations in each pixel of sample surface. This gives us a frozen in time realization of wavy sea surface. The same procedure gives us a sequence of surface realizations in a sequence of times $t_1, t_2, ..., t_L$. Using this method we can produce a real-time movies of the wavy sea surface and light reflected from a water body [1] with wavy surface.[2]

Computations

The values of sea surface orientation allow us to compute light reflection coefficient in each pixel of the sample surface. Averaging over $M \times M$ spatial and L temporal realizations gives as an average Fresnel reflection coefficient for each value of wind speed u and zenith angle θ. To compute actual values of Fresnel reflection coefficient we used the Pierson-Moskowitz (PM) [3] and Paul Hwang (PH) [4] spectrums.

The Pierson-Moskowitz wave energy spectrum is defined as:

$$\chi_{PM}(k) = \frac{0.00405}{k^3} \exp\left(-\frac{0.74\, g^2}{u^4 k^2}\right) \tag{1}$$

here $g = 9.8\ m/s$ is a gravitational acceleration, u is a wind speed in m/s, $k = g/u$ is a wave number in m^{-1}.

The Paul Hwang wave energy spectrum is defined as:

$$\chi_{PH}(k) = \frac{1 \cdot 10^{-4}}{k^2} \begin{cases} 5.45, \quad k < g u^2, \\ 1.74\, u \big/ \sqrt{k}, \quad g u^2 \le k < 16.0, \\ 6.96\, u/k, \quad 16.0 \le k < 100.0, \\ 0.682\, u \big/ \left(g + 0.00007\, k^2\right), 100 \le k \le 900, \\ 7.48 \cdot 10^6\, u/k^3, \quad 900 < k. \end{cases} \tag{2}$$

This spectrum is specifically tailored for remote sensing problems to produce correct values of mean square slopes of ocean waves.

The generated realizations of Fresnel reflection coefficients have been averaged over 100×100 pixels of sample sea surface areas and 80 time realizations to produce resulting angular distributions of Fresnel reflection coefficient. The values of elevations and orientations in each pixel have been obtained using one million computations, 1000 realizations for flat sine waves and 1000 significant points of wave number k in energy spectrum range. To generate random numbers we used Mersenne Twister random number generator [5-8] capable to produce evenly distributed random numbers in a cube of 626 dimensions. So each value of Fresnel reflection coefficient for any value of wind speed and zenith angle represents an average value of 800 billion individual computations.

Results

The main results of extensive computations for wind speed $u = 4\, m/\text{sec}$ are shown in Figures 1 and 2. Figure 1 shows results of our ray-tracing computations against values computed using Cox and Munk [9] wave slope distribution. For smaller values of reflection coefficient the difference is not significant, but for the larger values ($R_F \sim 0.7$) it can reach 20%. Because both ray-tracing and Cox and Munk implementations include corrections on shadowing effects, the

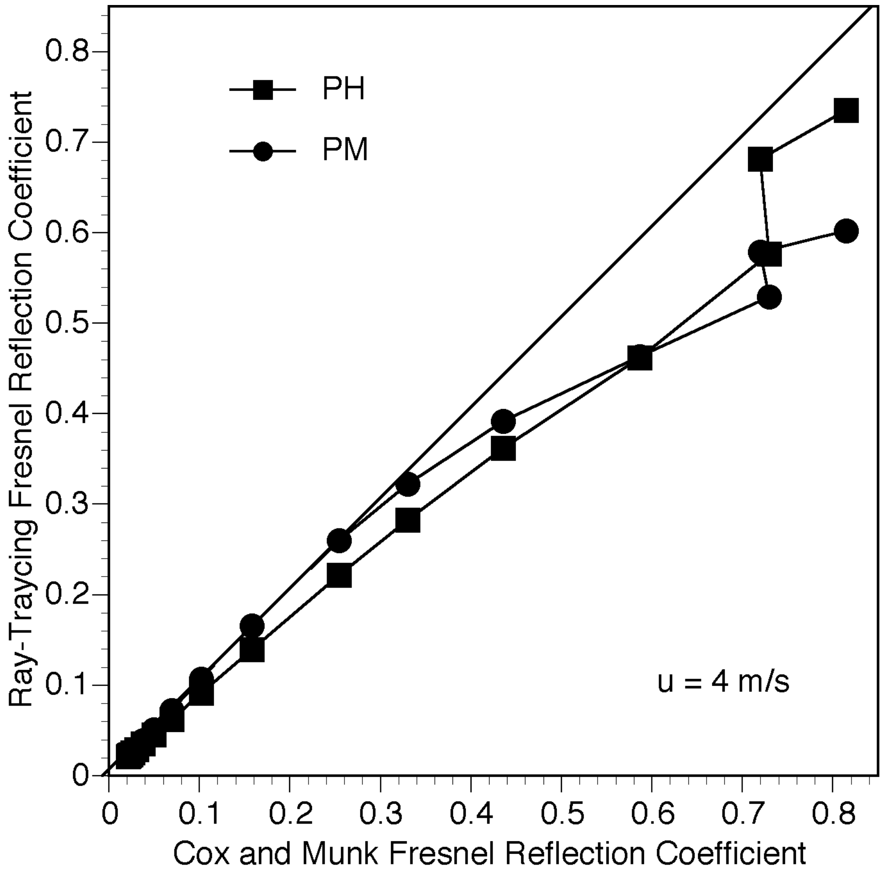

Figure 1. Ray-Tracing Fresnel reflection coefficient plotted against Cox and Munk [8] Fresnel Reflection Coefficient.

difference shown in the figure is due to the enhanced description of waves given by Eqs. (1) and (2). Figure 2 shows Fresnel reflection by wavy sea surface relative to the Fresnel reflection by the flat water surface. As it should be, for smaller zenith angles (smaller values of reflection) Fresnel reflection coefficient is slightly higher than the reflection coefficient of wavy surface. For larger zenith angles ($R_F > 0.5$) the reflection coefficient of flat surface exceeds Fresnel reflection of wavy surface.

Final equations

The results of extensive ray-tracing computations are compactly represented as a following regression equation,

$$R_F(\theta,u) = a_0(u) + R_F^0(\theta)\left\{a_1(u) + R_F^0(\theta)\left[a_2(u) + a_3(u)R_F^0(\theta)\right]\right\}, \tag{3}$$

where $R_F^0(\theta)$ is a Fresnel reflection coefficient of flat water surface,

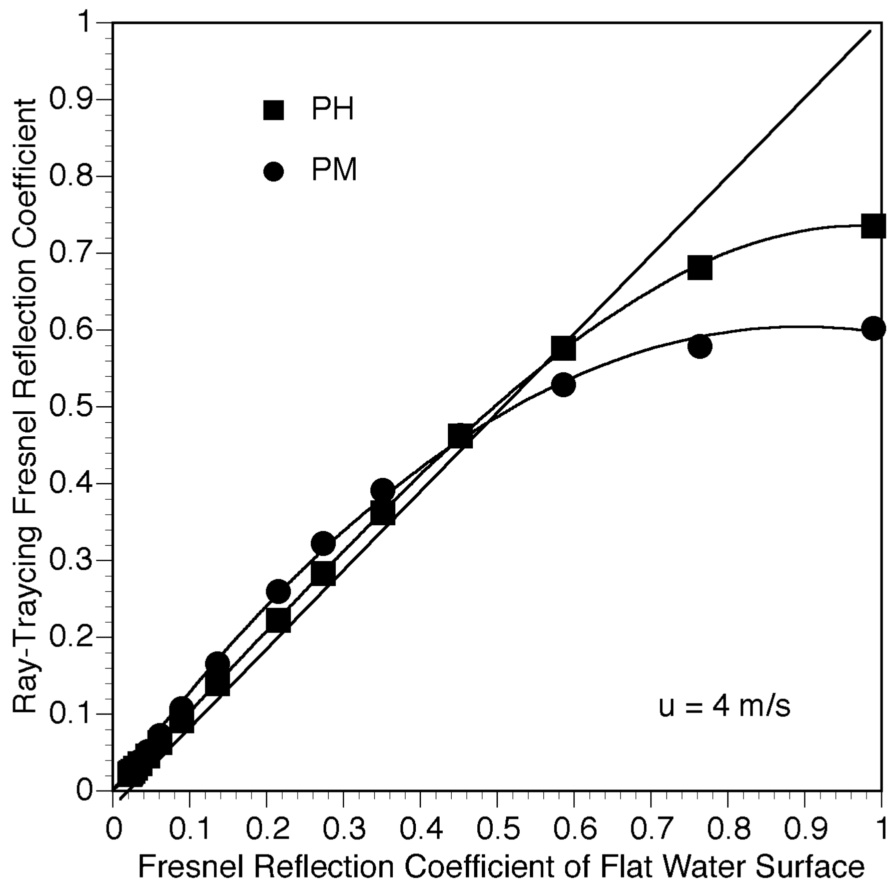

Figure 2. Example of ray-tracing Fresnel reflection coefficient from a rough water surface at wind speed equal to 4 m/s plotted against Fresnel reflection coefficient of flat water.

$$R_F^0(\theta) = \frac{1}{2}\left[\left(\frac{\cos\theta - \sqrt{n_w^2 - \sin^2\theta}}{\cos\theta + \sqrt{n_w^2 - \sin^2\theta}}\right)^2 + \left(\frac{n_w^2\cos\theta - \sqrt{n_w^2 - \sin^2\theta}}{n_w^2\cos\theta + \sqrt{n_w^2 - \sin^2\theta}}\right)^2\right], \quad (4)$$

here $n_w \approx 1.341$ is a refractive index of water, and θ is a solar zenith angle, and wind-speed-dependent coefficients $a_i(u)$ are given by the following equations:

$$a_0(u) \;=\; 0.001\left(6.944831 - 1.912076\,u + 0.03654833\,u^2\right), \quad r^2 \;=\; 0.9997, \quad (5)$$

$$a_1(u) \;=\; 0.7431368 + 0.0679787\,u - 0.0007171\,u^2, \quad r^2 \;=\; 0.9996, \quad (6)$$

$$a_2(u) \;=\; 0.5650262 + 0.0061502\,u - 0.0239810\,u^2 + 0.0010695\,u^3, \quad r^2 \;=\; 0.9995, \quad (7)$$

$$a_3(u) \;=\; -0.4128083 - 0.1271037\,u + 0.0283907\,u^2 - 0.0011706\,u^3, \quad r^2 \;=\; 0.9991. \quad (8)$$

Equations (3)-(7) for Fresnel reflection coefficient of wavy sea surface may be used for remote sensing and radiative transfer modeling.

Acknowledgments

The authors thank Paul A. Hwang for valuable consultations with water wave spectra. The authors from the Naval Research Laboratory (NRL) thank continuing support through the Hyperspectral Signatures (HS 8028-A1) program. This article represents NRL contribution NRL/PP/7330–01–0073.

Appendix: A FORTRAN code to compute Fresnel Reflection Coefficient of wavy water surface.

```
! *****************************************************************
      real function FresnWind(nWat,thet,u)
! -----------------------------------------------------------------
!   This function calculates Fresnel reflection coefficient of wavy
!   water surface.
!   This is the case of Paul Hwang wave energy spectrum distribution
! -----------------------------------------------------------------
!   nWat  = refraction index of water (nWat = 1.341)
!   thet  = angle (in degrees) at which light is incident on sea surface
!   u     = windspeed in m/sec, 0 <= u <= 12 m/s.
! *****************************************************************
      implicit   none
      real       nWat,thet,u
      real       phi,fr0,a0,a1,a2,a3, Fresnel

       a0 = ATAN(1.0)/45.0                ! = π/180
      phi = a0*thet                       ! converts angles to radians
      fr0 = Fresnel(nWat,phi)
       a0 = 0.001*(6.944831+u*(-1.912076+0.03654833*u))
       a1 = 0.7431368+u*(0.0679787-0.0007171*u)
       a2 = 0.5650262+u*(0.0061502+u*(-0.023981+0.0010695*u))
       a3 = -0.4128083+u*(-0.1271037+u*(0.0283907-0.0011706*u))
      FresnWind = a0+fr0*(a1+fr0*(a2+a3*fr0))

      return
      end

! *****************************************************************
      real function Fresnel(nWat,phi)
! -----------------------------------------------------------------
!   Computes Fresnel reflection coefficient of flat water surface
! -----------------------------------------------------------------
!   nWat  = refraction index of water
!   phi   = angle (in radians) at which light is incident on sea surface
! -----------------------------------------------------------------
!   aRef  = angle of refraction (in radians)
!   Rpar  = Fresnel refrlection coefficient for parallel polarization
!   Rper  = Fresnel refrlection coefficient for perpend. polarization
! *****************************************************************
      implicit   none
```

```
      real       nWat,phi, aRef,aDif,aSum,Rpar,Rper

      if (phi .ne. 0.) then
         aRef = ASIN(SIN(phi)/nWat)
         aDif = phi-aRef
         aSum = phi+aRef
         Rpar = TAN(aDif)/TAN(aSum)
         Rper = SIN(aDif)/SIN(aSum)
         fresnel = 0.5*(Rpar*Rpar+Rper*Rper)
      else
         aSum = (nWat-1.)/(nWat+1.)
         fresnel = aSum*aSum
      end if

      return
      end
! *********************************************************************
```

References

1. V. I. Haltrin, "Self-consistent approach to the solution of the light transfer problem for irradiances in marine waters with arbitrary turbidity, depth and surface illumination" *Appl. Optics*, **37**, 3773-3784 (1998).

2. V. I. Haltrin, W. E. McBride III, and A. D. Weidemann,"Fresnel reflection and transmission by wavy sea surface," in *Proceedings of the International Geoscience and Remote Sensing Symposium*, ed. Tammy I. Stein, pp. 1863-1865 (IEEE, Piscataway, NJ, USA, 2000).

3. W. J. Pierson Jr., and L. Moskowitz, "A proposed spectral form for the fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii," *J. Geophys. Res.*, **69**, 5181-519 (1964).

4. P. A. Hwang, "A study of a wavenumber spectra of short water waves in the ocean. Part II: Spectral model and mean square slope," *J. Atmos. Oceanic Technology*, **14**, 1174-1186 (1997).

5 M. Matsumoto and Y. Kurita, "Twisted GFSR generators," *ACM Trans. on Modeling and Computer Simulation*, **2**, 179-194, (1992)

6. M. Matsumoto and Y. Kurita, "Twisted GFSR generators II," *ACM Trans. on Modeling and Computer Simulation*, **4**, 254-266, (1994).

7. M. Matsumoto, and T. Nishimura, "Mersenne Twister: A 623-dimensionally equidistributed uniform pseudorandom number generator", *ACM Trans. on Modeling and Computer Simulation*," **8** (1), 3-30 (1998).

8. The C and FORTRAN codes of Mersenne Twister random number generator are available from <www.math.keio.ac.jp/~matumoto/emt.html>.

9. C. Cox, and W. Munk, "Measurements of the roughness of the sea surface from photographs of the sun's glitter," *Journ. Opt. Soc. America*, **44**, 838-850 (1954).